

Measuring the orientation and movement of marine animals using inertial and magnetic sensors - a tutorial

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Accelerometers and magnetometers are receiving increasing use in animal tags to measure orientation and activity. These sensors are compact, sensitive and consume little power making them suitable for use in tags on a wide range of taxa including fish, birds, turtles, seals, and whales, enabling a quantum leap in the precision with which animal movement is measured in the field. However, these sensors have limitations that are crucial to keep in mind in each step of the science process: from designing studies and tools, to interpreting data. The goal of this document is to derive a simple measurement model for accelerometers and magnetometers attached to animals. This model serves to highlight some of the potential sources of error in using these devices as well as opportunities to reduce these errors. It also proposes a standard nomenclature for the quantities measured by these sensors to aid in the exchange of data and findings. The focus here is on marine animals although the methods could be adapted readily to terrestrial and aerial animals.

Some simple mathematical concepts are required to describe the orientation and movements of animals in 3-dimensions. Accordingly this document begins with a short primer on linear algebra as used to define orientation. Measurement models for ideal accelerometers and magnetometers are then developed leading to a standard algorithm to estimate orientation. Non-ideal (but realistic) sensors are then considered and some robust estimators of orientation and activity are proposed. Finally, some methods to reduce errors using data-driven calibration are discussed. A glossary of terms is included at the end of the document as a starting point for developing a community-wide standard nomenclature.

1. Defining orientations and movements in 3-dimensions

Reference frames

Four mathematical concepts are needed to talk about the orientation of an animal: reference frames, basis matrices, rotation matrices and Euler angles. A reference frame is just a set of axes that we use to define the locations of a group of points. The numbers used to describe the positions of these points depend on where we center our reference axes and what direction they point in. For example, a reference frame could be defined as centered at 42° latitude, 0° longitude and sea-level (three numbers are required to define a point in 3-dimensions). The axes of the reference frame could point in the cardinal directions north, east and upwards. This kind of reference frame is called an earth-fixed or inertial frame and is used to describe the geographic movements of animals. In this frame, a position [1000,-10000,-500] meters would define a point with depth 500 m situated 1km north and 10km west of the point 42° N, 0° E.

Another type of reference frame is called a navigation frame. This frame is anchored to the animal (e.g., the center of the frame is at the center of gravity of the animal) but still has axes aligned with north, east and up. A position [10,5,-3] meters in the navigation frame defines a point 10 m north, 5 m east and 3 m below the center of the animal, wherever it is. The navigation frame is used to define the orientation of an animal and its movements, e.g., its velocity, in an earth-relative way. Note that there

are several ways that the axes can be defined in a reference frame. The north-east-up axes used here follow the left-hand rule, a mnemonic based on the left-hand index finger pointing north, the 2nd finger pointing east, and the thumb pointing up. Other protocols for the axes (e.g., the right-hand rule) can be used as long as the axes are defined consistently over all the reference frames (i.e., all axes should be left- or right-handed).

A third reference frame is called the animal frame. Like the navigation frame, this frame is anchored to the animal but, unlike the navigation frame, the axes of the animal frame are aligned with the cardinal axes of the animal, i.e., the caudo-rostral, left-right, and ventro-dorsal axes. Both the navigation and animal frames follow the animal as it moves but the animal frame also follows the orientation of the animal. Thus the difference between the navigation and animal frames defines the orientation of the animal (the precise meaning of the word 'difference' will be defined below).

A final reference frame, called the tag frame, may be needed to work with tag data from animals. This frame accounts for the orientation of the tag on the animal with respect to the animal frame. The tag frame is centered on the animal as with the navigation and animal frames but the axes are aligned with the measurement axes of sensors in the tag. If an animal is restrained for tag deployment and the tag is rigidly attached, it may be possible to align the tag axes with the animal axes. In this case, the tag and animal frames will coincide and data analysis will be simplified. However, some tags are attached wherever they fit on the animal or wherever they happen to go on and the difference between the tag axes and animal axes must be accounted for in data analysis. The difference between the tag and animal frames is determined by the orientation of the tag on the animal. Note that we are ignoring the fact that tags are not usually attached at the center of gravity of an animal so the tag and animal frames are actually offset from each other. It is not yet clear if this is an important issue but to avoid the problem, the navigation, animal and tag frames could all be considered to be anchored to the center of the tag rather than the animal so that they coincide.

Basis and rotation matrices

Mathematically, directions in 3-dimensional space can be described by vectors with 3 numbers arranged in a column. Reference frames contain three axes and so are defined by 3 vectors, each describing one of the axes. The three vectors are put together into a 3x3 matrix which is called a basis. For example, the basis for the left-hand navigation frame is $\Psi^n = [N \ E \ U]$ where N, E and U are each 3x1 vectors describing the directions north, east and up. Throughout this document, we will use capital letters to denote vectors and bold capitals for matrices. Single-dimensional (scalar) quantities will have small letters. The super-script ⁿ in Ψ^n is used to signify that this basis matrix defines the navigation frame. It is not important precisely how directions such as 'north' are represented by a vector - these directions will cancel out in the mathematics at a later stage. Basis matrices are orthogonal meaning that each direction vector has a magnitude of 1 and the vectors are perpendicular to each other. An example of a simple orthogonal matrix is the 3x3 identity matrix $I = [1 \ 0 \ 0; 0 \ 1 \ 0; 0 \ 0 \ 1]$ and you could use this for Ψ^n if it helps to be specific. In this case, north is represented by the vector $N = [1 \ 0 \ 0]^T$ which is a short-hand for saying that N is a 3x1 vector with elements 1, 0, and 0 (the ^T means 'transpose' which is a vector or matrix operation that swaps row and columns).

A basis for the animal frame is defined in the same way, as the 3x3 orthogonal matrix $\Psi^a = [X^a, Y^a, Z^a]$ where X^a is a unit vector defining the animal's longitudinal axis (caudal-rostral), Y^a the transverse axis (left-right) and Z^a the dorsal-ventral axis. The superscript ^a is used to denote the animal frame. Because the animal and navigation frames are anchored at the same point, the only difference between them is

that the axes point in different directions. It is therefore possible to rotate one frame so that it matches the other. Rotations in 3-dimensions are expressed by 3x3 matrices called rotation matrices (also known as direction cosine matrices). Rotation of a basis describing one reference frame to form another one is achieved by multiplying the basis by a rotation matrix. This means that there is a rotation matrix, \mathbf{Q} , which rotates the navigation frame to match the animal frame, i.e., $\Psi^a = \Psi^n \mathbf{Q}$. As the only difference between these frames is due to the orientation of the animal, the 3x3 matrix \mathbf{Q} defines the animal's orientation in 3-dimensional space.

Like bases, rotation matrices are orthogonal. The product of two rotation matrices is also a rotation matrix (it makes a rotation equal to the sum of the two rotations). The inverse of a rotation matrix, \mathbf{Q} , is denoted by \mathbf{Q}^{-1} , which is also a rotation matrix. It defines a rotation that reverses or 'undoes' the original rotation. This is because $\mathbf{Q}\mathbf{Q}^{-1} = \mathbf{I}$, for any matrix \mathbf{Q} for which the inverse exists, and multiplication by \mathbf{I} means 'no change'. So the relationship $\Psi^a = \Psi^n \mathbf{Q}$, relating the navigation frame to the animal frame, can be expressed in another way: if we multiply both sides by \mathbf{Q}^{-1} we get: $\Psi^a \mathbf{Q}^{-1} = \Psi^n \mathbf{Q}\mathbf{Q}^{-1}$ (remember that, when multiplying matrices, order matters, so $\Psi^n \mathbf{Q}$ is not the same as $\mathbf{Q}\Psi^n$). This means that $\Psi^a \mathbf{Q}^{-1} = \Psi^n$. so \mathbf{Q}^{-1} defines the rotation from the animal to navigation frames.

In the same way that Ψ^a and Ψ^n are related by a rotation matrix, the animal and tag frames are related by another rotation matrix, \mathbf{W} . This matrix defines the orientation of the tag on the animal. The basis for the tag frame can be written as $\Psi^t = [X^t, Y^t, Z^t]$, where X^t , Y^t , and Z^t define the sensitive axes of the sensors in the tag (to help visualize the tag frame, you could imagine that these axes line up with the physical axes of the tag, e.g., back-front, left-right, and bottom-top in the left-hand rule). Using these definitions, we can write the relationship between the tag and animal frames as: $\Psi^a = \Psi^t \mathbf{W}$ or $\Psi^a \mathbf{W}^{-1} = \Psi^t$. Rotations can be combined by multiplying their rotation matrices, so the relationship between the navigation frame and the tag frame is $\Psi^t = \Psi^n \mathbf{Q}\mathbf{W}^{-1}$. The combined rotation $\mathbf{Q}\mathbf{W}^{-1}$ is itself a rotation matrix that defines the orientation of the tag with respect to the navigation frame, i.e., the result of the animal's orientation in space and the orientation of the tag on the animal. Note that, if the tag is aligned with the animal axes, $\mathbf{W} = \mathbf{I}$.

Defining orientation

The rotation matrix, \mathbf{Q} , describes the orientation of the animal in space at a moment in time. As the orientation changes with time, the matrix \mathbf{Q} also changes with time and we can write \mathbf{Q}_t when we want to denote the particular value of \mathbf{Q} at a time t ¹. Although \mathbf{Q}_t is a full description of the orientation at time t , it is not very easy to interpret. There are 9 numbers in the \mathbf{Q} matrix but only 3 numbers are really needed to describe any orientation. One way to do this is with the Euler angles, pitch, roll and yaw, defined as follows:

yaw (γ): rotation around the local Z axis (like the body motion of a swimming tuna or seal).

pitch (p): rotation around the local Y axis (like the body motion of a swimming whale).

roll (r): rotation around the local X axis. This could define a 'corkscrew' type of motion.

where 'local axis' refers to one of the cardinal axes of the starting reference frame. Any orientation can be described as being composed of these elemental rotations. The order of rotations matter: the sequence often used is yaw then pitch then roll, and we will use this sequence here. The rotation matrices corresponding to these rotations are:

¹ We are starting to get a few uses of t here. A sub-script _{t} means time. A super-script ^{t} means tag-frame and a super-script ^{T} means transpose.

$$Y(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P(p) = \begin{bmatrix} \cos(p) & 0 & -\sin(p) \\ 0 & 1 & 0 \\ \sin(p) & 0 & \cos(p) \end{bmatrix}$$

$$R(r) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(r) & -\sin(r) \\ 0 & \sin(r) & \cos(r) \end{bmatrix}$$

No matter what the orientation of the animal, it can be expressed as certain amounts of yaw, pitch and roll. This means that \mathbf{Q}_t can be expressed as the product of the three rotation matrix corresponding to the Euler angles, i.e., $\mathbf{Q}_t = Y(\gamma_t)\mathbf{P}(p_t)\mathbf{R}(r_t)$. Thus the animal and navigation frames are related as:

$$\Psi^a = \Psi^n \mathbf{Q}_t = \Psi^n Y(\gamma_t)\mathbf{P}(p_t)\mathbf{R}(r_t)$$

There is no loss of generality in constraining one of the angles of rotation (here we choose pitch) to the range $[-90^\circ, 90^\circ]$ and the other two to the range $[-180^\circ, 180^\circ]$ ². An orientation with $p > 90^\circ$ can also be expressed using the angles:

$$\gamma' = \gamma + 180^\circ, p' = 90^\circ - p, r' = r + 180^\circ$$

which gives $p < 90^\circ$. By constraining p to the range $[-90^\circ, 90^\circ]$ we avoid one source of ambiguity in describing orientation. Each triple $\{\gamma, p, r\}$ defines a unique orientation with respect to a reference frame, with the exception of when $|p| = 90^\circ$ (e.g., when the animal is oriented vertically upwards or downwards). In these orientations, the roll and yaw rotations are identical and only the sum $r + \gamma$ is relevant (this situation is termed 'gimbal-lock' in navigation).

We have used the term yaw here to describe a rotation around the local vertical axis. When describing an animal's orientation with respect to the navigation frame, a yaw involves a change in the compass heading of the animal. 'Heading' refers to the pointing angle of the animal with respect to true north and is the correct term to use in defining an animal's orientation. Thus the 3 numbers defining an animal's orientation with respect to the navigation frame are $\{h, p, r\}$. The triple used to define a change in orientation, howsoever the animal is currently oriented, is $\{\gamma, p, r\}$ with the angles defined with respect to the current animal frame. Distinct terms like heading and yaw are not available for the pitch and roll rotations so the same word is used to describe orientations with respect to the navigation frame or the current animal frame, e.g., an animal could be described as having a pitch of -30° with respect to the horizontal plane (i.e., it is pointing nose-down by 30°) but it could also then perform a pitching motion (i.e., rotation around its left-right axis) while swimming with an extent of $\pm 10^\circ$. The first pitch angle is defined with respect to the navigation frame while the second is defined with respect to the initial orientation of the animal. This dual use of the same term can cause some confusion if the movements

² When defining ranges, a square bracket [means greater than or equal while a round bracket (means strictly greater than.

are not carefully defined.

Euler rotations are a compact and intuitively appealing way to define orientations especially given their similarity to locomotory movements of some animals. However, they require careful definition of the sequence and directions of rotations, as well as how gimbal-lock is handled. Visualization and mathematical analysis of Euler angles is also complicated. Plots of pitch angle are straightforward to interpret for diving animals, but plots of heading and roll for active animals frequently contain lots of jumps between ± 180 as the animal moves back and forth between eastward and westward headings or as it performs continuous upside down rolls. It is hard for the eye to interpret motions across these jumps in plots and it is especially dangerous to filter or interpolate heading and yaw data as this introduces artefactual intermediate points in the jumps. In general, Euler angles should never be filtered - the filter should be applied instead to the raw sensor measurements from which they are derived. Statistical analysis of Euler angles should also be made with care and may require circular statistics. To avoid some of these problems, other ways of defining orientations are sometimes used, especially in the fields of computer graphics and virtual reality. Quaternions and angle-axis representations, which use 4 parameters to define orientation, are common in these fields and good descriptions of these can be found on the internet. Nonetheless, Euler angles are likely to remain the usual way to express animal orientations and movements and so it is important to keep in mind the limitations of this representation.

Having defined a basic framework for describing animal orientations, we now turn to how orientation and movement can be measured using on-animal sensors.

2. Sensing animal orientation and movement

Estimating the orientation and movement of a platform from measurements made on the platform itself is a problem that has been thought about since at least the early days of sea-faring. In recent times, the problem has been actively studied in the context of airplanes, space vehicles and underwater autonomous vehicles. The objective has been to control the orientation (also called the 'attitude' in navigation parlance) of the vehicle and estimate its track. Our objectives in animal studies are slightly different: we wish to observe the orientation and movements of the animal to infer its behavior and energetics, but the methods are much the same. The core sensor suite used to navigate a vehicle is termed an inertial navigation sensor (INS) and comprises a tri-axial gyroscope and a tri-axial accelerometer. A gyroscope measures the turn-rate in its sensitive axis, i.e., the change in yaw, pitch or roll per second. Three gyroscopes aligned to each of the three platform axes (termed a tri-axial gyroscope) are needed to cover all three Euler angles. The orientation can be estimated by integrating the outputs of the tri-axial gyroscopes, i.e., integrating pitch-rate to pitch etc. The gyroscope and accelerometer signals are then combined to deduce movement (Grewal et al. 2001). Until recently, gyroscopes were too large and power hungry to use in animal tags. This situation is rapidly changing and some compact, low-power 2- and 3-axis gyroscopes are now available making it possible to start thinking about INS systems for animals (Martin et al. 2005). This will likely be a hot topic of research in the next few years.

Accelerometers

In the absence of gyroscopes, most animal tags have focused on the use of accelerometers and magnetometers to deduce orientation (Johnson et al. 2003, Mitani et al. 2003). An accelerometer measures the acceleration in its sensitive axis. It consists of a mass connected to a force sensor: when the device is accelerated forward, the inertia of the mass pushes back against the force sensor creating a

signal. Three accelerometers aligned to the animal's axes (called a tri-axial accelerometer) are needed to measure the total acceleration experienced by the animal. Tri-axial accelerometers are extremely compact (e.g., $<8 \text{ mm}^3$), inexpensive and take very little power ($<1\text{mW}$), accounting for their popularity in animal tags. Their bandwidth can exceed 1 kHz allowing even very rapid movements to be sampled.

Accelerometers measure the total force experienced by the internal mass (called the proof mass). This comprises two components: (i) the force caused by the inertial reaction of the mass to the movement of the platform that the sensor is attached to, and (ii) the downwards force due to gravity. Both forces are turned into acceleration by dividing by the mass (remember that $f=ma$). The first component is called the specific acceleration (the term 'dynamic acceleration' is also used but this should be avoided as it is ambiguous). The second term can be called the gravity-related acceleration (the term 'static acceleration' is sometimes used but this is very misleading as the value measured is far from static changing whenever the animal changes orientation). It is crucial to appreciate that all accelerometers always measure these two components of acceleration, and that the specific and gravity-related terms can not be separated in the output signal of an accelerometer without making some strong assumptions about the way the animal moves, as will be discussed later.

Most tri-axial accelerometers are etched from a tiny plate of silicon in a process called MEMS (micro-electro-mechanical system) fabrication. The proof mass is suspended on elastic anchors and it moves a little when accelerated. This movement is measured and converted into a voltage. To illustrate the two different types of acceleration that the sensor measures, imagine that the sensor is held stationary with the mass pointing down. The mass will want to fall towards the earth and so will stretch its anchors creating a voltage. This is the gravity-related acceleration output. If the sensor is then accelerated upwards, the mass will resist this motion and produce an additional output proportional to the platform acceleration.

The magnitude of the gravity-related acceleration component in each of the sensor axes depends on the orientation of the sensor. The pitch and roll (but not the heading) of the sensor with respect to the navigation frame can thus be estimated from the accelerometer outputs and we will derive the equations for this shortly. Some very sensitive single axis accelerometers use the piezoelectric effect to measure force. These differ from MEMS devices in that they do not have a frequency response that extends to 0 Hz and so have no output when stationary. In effect, they are like a MEMS sensor with a high-pass filter. These sensors still measure both acceleration components but filter the combined output making them unsuitable for estimating orientation. The value of this kind of filtering will be discussed later.

Estimating pitch and roll

An accelerometer in a stationary tag (or a tag moving at a constant speed in a constant direction) measures the strength of the gravitational field in its sensitive axis. If the sensitive axis of the accelerometer coincides with the tag's X^t axis, the accelerometer measurement a_x , will be equal to $a_x = -gU^T X^t$, where U is the up-vector in the navigation frame and g is the gravitational field intensity which is 9.8m/s^2 at the earth's surface³. The term $-gU$ is the gravity vector in the navigation frame, i.e., a vector with magnitude $g \text{ m/s}^2$ pointing downwards. The vector multiplication $U^T X^t$ is called the

³ In fact, g depends to a minor degree on location and altitude. Using a value of 9.8ms^{-2} for g will incur an error of about $+3 \text{ mg}$ at the poles, -2mg at the equator, and -0.3 mg per km of dive depth [ref]. But these errors of less than $\pm 0.5\%$ are negligible when compared to the other sources of error discussed here.

'projection' of U on X^t , and is the amount that the X^t axis extends in the direction U given the current orientation of the tag. We know gU and we are measuring a_x so the equation $a_x = -gU^T X^t$, tells us something about how the X^t axis is oriented in space.

Repeating the above step with each of the three accelerometer axes, aligned with the tag axes X^t , Y^t , and Z^t , gives:

$$a_x = -gU^T X^t, \quad a_y = -gU^T Y^t, \quad a_z = -gU^T Z^t,$$

These 3 accelerometer outputs can be combined into a 1×3 vector, $A^t = [a_x, a_y, a_z]$. We use a row vector for A^t instead of a column vector because it simplifies the mathematics: we will use row vectors for all sensor measurements but remember that the axes in the reference frames (U , X^t etc.) are all column vectors. Combining the definitions for each accelerometer axis we get:

$$A^t = -gU^T [X^t \ Y^t \ Z^t] = -gU^T \Psi^t$$

Using the relationship between the tag and navigation frame, derived above, we get: $A^t = -gU^T \Psi^n Q W^{-1}$. From the definition of the navigation frame, the term $U^T \Psi^n$ is equal to $U^T [N \ E \ U]$ which equals $[0 \ 0 \ 1]$ because U is perpendicular to N and E (i.e., $U^T N = U^T E = 0$), and U has unit length so that $U^T U = 1$. So:

$$A^t = -g.[0 \ 0 \ 1] Q W^{-1}$$

Thus the tri-axial accelerometer measurement in a stationary tag is dependent on the orientation of the animal, Q , and the orientation of the tag on the animal, W .

To simplify things, we consider first the case that the tag is perfectly aligned with the animal's axes. Then $W = I$ and $A^t = A^a = -g.[0 \ 0 \ 1] Q$, where A^a is the acceleration in the animal frame, i.e., the total acceleration experienced by the animal. The term $[0 \ 0 \ 1] Q$ is just the bottom row of Q . Using the definition of Q in terms of yaw, pitch and roll, this bottom row is $[0 \ 0 \ 1] Q = [0 \ 0 \ 1] Y(\gamma) P(p) R(r)$. Looking at the structure of the yaw rotation matrix, it is apparent that $[0 \ 0 \ 1] Y(\gamma) = [0 \ 0 \ 1]$ and so:

$$A^a = -g.[0 \ 0 \ 1] P(p) R(r)$$

This means that the accelerometer signals measured on a stationary animal are only dependent on the animal's pitch and roll which is reasonable: an accelerometer cannot measure heading as the gravity vector has no horizontal component.

Substituting the definitions of the pitch and roll rotation matrices, we get:

$$A^a = -g.[\sin(p) \ \cos(p)\sin(r) \ \cos(p)\cos(r)]$$

Given this, the pitch and roll of the animal can be estimated from A^a with a little algebra as follows:

$$\hat{p} = -\text{asin}(a_x^a / g) \quad , \quad \hat{r} = \text{atan}(a_y^a / a_z^a)$$

where the circumflex over the p and r indicates that these are estimates of the actual p and r . Thus, the

animal's pitch and roll can be deduced from simple trigonometric functions of the acceleration measurements made on the animal.

The magnitude of a vector is given by the square-root of the sum of the squares of each component, i.e., $\|A^a\| = \sqrt{(a_x^a)^2 + (a_y^a)^2 + (a_z^a)^2}$, where $\|\cdot\|$ denotes the magnitude or norm of a vector⁴. Applying this to the above equation (which, remember, is only true in the absence of specific acceleration), $\|A^a\| = g$. So far we have assumed that $\mathbf{W}=\mathbf{I}$ but no matter what \mathbf{W} is, $\|A^a\|$ always equals $\|A^t\|$. This is because a rotation does not change the magnitude of a vector, just its angle. So the pitch estimator can also be written as:

$$\hat{p} = -\text{asin}(a_x^a / \|A^t\|)$$

This estimator is more robust to calibration errors in the accelerometer, e.g., errors in the scale factor. The roll estimator, being a ratio of two accelerations is already robust to scale factor errors. However, the roll estimate is not robust to errors and noise at high absolute pitch angles (i.e., a nearly vertical up or downwards animal) because $\cos(p)$ will then be small making a_z^a also small. Noise and errors in the accelerometer signals will be divided by a small number (i.e., multiplied by a large number) in the roll estimator leading to large roll errors at high pitch angles. As a result, roll errors could be 6 times larger when $|p| = 80^\circ$ than when the animal is horizontal.

The above pitch and roll estimators require that the tag is perfectly aligned with the body axes of the animal. This is often not the case and so it is necessary to first translate the acceleration vector measured by the tag, A^t , to the vector that would be measured if the tag was aligned perfectly on the animal, A^a . The acceleration vector measured by the tag was derived above as: $A^t = -g \cdot [0 \ 0 \ 1] \mathbf{Q} \mathbf{W}^{-1}$. while the acceleration vector that would be measured in the animal frame is $A^a = -g \cdot [0 \ 0 \ 1] \mathbf{Q}$. Substituting the second equation into the first, we get that $A^a = A^t \mathbf{W}$, i.e., we can predict the acceleration that would be measured by a perfectly aligned tag simply by multiplying the actual acceleration vector by the rotation matrix, \mathbf{W} . The resulting vector can then be used to estimate the animal's pitch and roll using the above formulae.

The rotation matrix, \mathbf{W} , is dependent on the orientation of the tag on the animal which, for a rigid tag attachment, should be constant. In practice, \mathbf{W} may change occasional due to movement of the tag caused by rubbing or slippage. Methods to estimate \mathbf{W} based on the measured data will be discussed later. Photographs of tags on animals are also a useful source of information to estimate \mathbf{W} or, at least to perform sanity checks on automatic estimates of \mathbf{W} .

Magnetometers

To measure heading, a magnetometer or magnetic-field sensor is required. A magnetometer measures the strength of the earth's magnetic field in the direction of its sensitive axis. Three magnetometers aligned with the animal's axes (termed a tri-axial magnetometer) are needed to measure the entire field at the animal. Magnetometers are usually made from magnetic-sensitive resistors arranged in a type of circuit called a Wheatstone bridge. Bridge sensors are very accurate but require some power to drive the bridge so magnetometers are quite power hungry (e.g., 20mW) and are often turned off in between measurements to save power. Magnetometers are fairly inexpensive and small (25 mm³) but require some care and maintenance as will be discussed later. Magnetometers produced in the USA may be

⁴ This measure of vector magnitude is also called the 2-norm or just the norm.

subject to export restrictions related to the ITAR (International Trafficking in Arms Regulations) legislation.

Estimating heading

As with an accelerometer, a magnetometer measures the projection of the earth's magnetic field on its sensitive axis. If the sensitive axis is X^t , for example, the measured magnetic field strength is $m_x = B^T X^t$, where B is the magnetic field vector at the location of the sensor. The earth's magnetic field is more complex and variable than its gravitational field. At any location, B can be expressed in the left-hand navigation frame axes as:

$$B = b.(\cos(i).\cos(d).N + \cos(i).\sin(d).E - \sin(i).U)$$

where b is the local magnetic field intensity in micro-Tesla (μT), i is the local inclination angle (by convention, the inclination is positive if B points below the horizontal as it does in most of the northern hemisphere), and d is the local declination angle (a positive declination means that magnetic north is east of true north). Because B is the weighted sum of all of the navigation frame axes N , E and U , it doesn't usually coincide with any one of these, i.e., in most parts of the world, the magnetic field vector does not point directly towards true north, instead it points d degrees to the east of north and i degrees down. The parameters $\{b, i, d\}$ for a given latitude and longitude can be obtained from magnetic field charts and magnetic field calculators on the internet. In open areas, B can be considered roughly constant over areas of many square kilometers. For heading estimation, an important parameter is the magnitude of the horizontal component of B as this determines the strength of the compass response and so the accuracy of the compass. The horizontal field strength is $b\cos(i)$ which varies widely over the earth's surface, from $40\mu T$ in Malaysia to 0 at the magnetic poles. For any magnetic compass, heading errors will be large near the magnetic poles and small at the equator.

Combining the signals from each axis of a tri-axial magnetometer into a vector, $M^t = [m_x, m_y, m_z]$, gives: $M^t = B^T \Psi^t$ where Ψ^t is the basis matrix of the tag frame. Applying the relationship between the tag and navigation frames, we get: $M^t = B^T \Psi^n Q W^{-1}$.

As with the accelerometer, the vector, M^a , that would be measured by a sensor aligned with the animal's axes is obtained by multiplying M^t by W , the rotation matrix defining the orientation of the tag on the animal, i.e., $M^a = M^t W$. Applying this to the formula above for M^t and substituting the yaw, pitch and roll expansion of Q , gives:

$$M^a = B^T \Psi^n Q = B^T \Psi^n Y(\gamma) P(p) R(r)$$

Thus M^a depends in a complicated way on the pitch, roll and heading of the animal and these elemental rotations cannot be deduced unambiguously from magnetometer signals alone.

You may have noticed the older-style compasses in boats. These contain a ball with a horizontal magnet inside and a weight at the bottom. The ball floats in a fluid and so always orients so that the weight is downwards no matter how the boat is tilted. This ensures that the magnet is horizontal and so will turn the ball to align with the earth's magnetic field. The process of keeping the magnet horizontal in a compass is called 'gimbaling' and it is used to avoid the dependence of the compass reading on the platform pitch and roll. The magnetometer in a tag is not mechanically gimballed and so this operation must be performed in post-processing. Using the estimates of pitch and roll from the accelerometer, we

can estimate the rotation matrices, $\mathbf{P}(p)$ and $\mathbf{R}(r)$ and use the inverse of these to reverse the pitch and roll rotations on the magnetometer reading. Assuming that \hat{p} and \hat{r} are correct, i.e., $\hat{p}=p$ etc., the gimballed magnetometer vector is:

$$\mathbf{M}^a(\mathbf{P}(p)\mathbf{R}(r))^{-1} = \mathbf{B}^T \mathbf{\Psi}^n \mathbf{Y}(\gamma) \mathbf{P}(p) \mathbf{R}(r) (\mathbf{P}(p) \mathbf{R}(r))^{-1}$$

The inverse of the product of two matrices is the product of the inverses with the order reversed so $(\mathbf{P}(p)\mathbf{R}(r))^{-1} = \mathbf{R}(r)^{-1} \mathbf{P}(p)^{-1}$. Remembering that $\mathbf{R}\mathbf{R}^{-1} = \mathbf{I}$ for any invertible matrix, we get:

$$\mathbf{M}^a \mathbf{R}(r)^{-1} \mathbf{P}(p)^{-1} = \mathbf{B}^T \mathbf{\Psi}^n \mathbf{Y}(\gamma)$$

Thus, by multiplying the magnetometer vector first by \mathbf{W} to get to the animal frame, and then by the inverses of the roll and pitch rotation matrices, we get the reading that we would have made with a gimballed magnetometer, i.e., a magnetometer in a horizontal frame of reference attached to the animal. We call this derived reading \mathbf{M}^h to signify that it comes from the horizontal frame, i.e., $\mathbf{M}^h = \mathbf{M}^a \mathbf{R}(r)^{-1} \mathbf{P}(p)^{-1}$. From the above equation, $\mathbf{M}^h = \mathbf{B}^T \mathbf{\Psi}^n \mathbf{Y}(\gamma)$, i.e., \mathbf{M}^h is dependent on the heading of the animal and the local magnetic field parameters but is independent of pitch and roll. Substituting the definition of \mathbf{B} into this equation and remembering that $\mathbf{N}^T \mathbf{N} = \mathbf{I}$ but $\mathbf{N}^T \mathbf{E} = \mathbf{N}^T \mathbf{U} = 0$, we get that:

$$\mathbf{M}^h = b. [\cos(i)(\cos(\gamma).\cos(d + \sin(\gamma)\sin(d)) , -\cos(i)(\sin(\gamma).\cos(d - \cos(\gamma)\sin(d)) , -\sin(i)]$$

That is \mathbf{M}^h , which we get by rotating the measured magnetometer vector by $\mathbf{W}\mathbf{R}(r)^{-1}\mathbf{P}(p)^{-1}$, is a function of the yaw of the animal and the parameters of the magnetic field vector. As \mathbf{B} points towards magnetic north, the value of yaw, γ , that solves this vector equation is the animal's magnetic heading. To get the animal's heading with respect to true north, we define $h=\gamma-d$ and use some trigonometric identities to get the simpler result:

$$\mathbf{M}^h = b. [\cos(i).\cos(h) , -\cos(i).\sin(h) , -\sin(i)]$$

An estimate of the heading with respect to true north is then given by: $\hat{h} = \text{atan}(-m_y^h / m_x^h)$, where m_y^h is the 2nd element of the \mathbf{M}^h vector etc.

To summarize, the processing steps required to estimate an animal's orientation are as follows:

1. Estimate the orientation of the tag on the animal and calculate the rotation matrix \mathbf{W} . This needs to be done only once per deployment or whenever the tag moves on the animal.

For each sample time:

2. Measure the accelerometer and magnetometer vectors, \mathbf{A}^t and \mathbf{M}^t , in the tag frame.

3. Multiply \mathbf{A}^t by \mathbf{W} to get \mathbf{A}^a .

4. Estimate pitch and roll using:

$$\hat{p} = -\text{asin}(a_x^a / \|A^a\|) , \hat{r} = \text{atan}(a_y^a / a_z^a)$$

5. Multiply \mathbf{M}^t by \mathbf{W} to get \mathbf{M}^a .

6. Compute the rotation matrices $\mathbf{P}(\hat{p})$ and $\mathbf{R}(\hat{r})$ from \hat{p} and \hat{r} .

7. Multiply \mathbf{M}^a by $\mathbf{R}(\hat{r})^{-1} \mathbf{P}(\hat{p})^{-1}$ to get \mathbf{M}^h .

8. Estimate the heading using $\hat{h} = \text{atan}(-m_y^h / m_x^h)$.

Note that the matrix inverses can all be replaced by transposes because $\mathbf{Q}^{-1}=\mathbf{Q}^T$ if \mathbf{Q} is an orthogonal matrix such as a rotation matrix. This reduces the computational requirements of the algorithm.

The dead-reckoned track

One of the reasons for measuring the orientation of an animal is to estimate its track. While terrestrial animals can be tracked with GPS sensors in a tag, this is not possible for animals that spend much of their time submerged. Dead-reckoning is a widely used, but much less accurate, way to estimate the tracks of aquatic animals (Wilson & Wilson 1988, Wilson et al. 2007). In dead-reckoning, the displacement of an animal from a starting point, T_0 , is estimated by integrating the velocity vector, V_t , of the animal, i.e.:

$$\hat{T}_{t_1} = \int_{t_0}^{t_1} V_t dt$$

where t_0 and t_1 are the start and end times of the interval, respectively, and \hat{T}_{t_1} is the estimated position of the animal at t_1 . Here, T_0 is a 3x1 vector defining the origin of a local-level earth-fixed reference frame and \hat{T}_{t_1} is an estimate of the position of the animal at time t_1 in this reference frame. V_t is the velocity of the animal defined in the navigation frame.

We rarely know the velocity vector of the animal - this would require a three-dimensional speed sensor on the animal as well as information about the water currents at the depth of the animal. We may, however, have an idea of the average forward speed of the animal. This could come from a turbine or paddle speed sensor on the tag (Blackwell et al 1999), or could be derived from the flow-noise recorded by the tag (Goldbogen et al. 2006) or the change in depth of the animal (Zimmer et al. 2003). To estimate the velocity vector, we need to know the direction of movement of the animal, i.e., the course-made-good in navigation parlance. For swimming and flying animals, the direction of movement should be, on average, fairly close to the longitudinal (caudo-rostral) axis of the animal, X^a . It will not be identical because of the effects of lift, buoyancy, drag, and the way that thrust is developed by the animal. It could also be very different during turns, especially when the animal's body is itself bending making the definition of X^a uncertain. Nonetheless, in lieu of anything better, V_t can be estimated by the product $s_t X^a$, where s_t is the speed at time t , and this can be used to predict the track taken by the animal. Knowing the pitch, roll and heading of an animal, X^a can be estimated from:

$$\hat{X}^a = [1 \ 0 \ 0] \mathbf{Y}(\hat{h}) \mathbf{P}(\hat{p}) \mathbf{R}(\hat{r})$$

As sensor data is invariably recorded digitally, the integration can be replaced with summation, and the dead-reckoned animal position at sensor sample k can be computed iteratively from the position at sample $k-1$ by the formula:

$$\hat{T}_k = \hat{T}_{k-1} + (1/f_s) s_k \hat{X}_k^a$$

where f_s is the sensor sampling rate in Hz, and s_k is an estimate of the animal's forward speed in m/s at time k/f_s . There are numerous sources of error with dead-reckoned tracks and these must be treated with caution (Wilson et al. 2007). Because errors are summed throughout the track, the absolute

localization error increases with time and can reach hundreds of meters over a long dive of a marine animal.

Non-ideal sensors

The above analysis makes several assumptions. First, we assumed that the animal was stationary or moving with a constant velocity (i.e., constant speed and heading). This was to avoid any specific acceleration term in the accelerometer signal. The second assumption was that the sensors are perfectly calibrated, i.e., the accelerometer and magnetometer outputs are in units of m/s^2 and μT with no scale or offset errors. Thirdly we assumed implicitly that the sensors had sufficient bandwidth to track the animal's movements and that there were no digital sampling issues. Finally, we assumed that the rotation matrix, \mathbf{W} , can be deduced from the data but we didn't say how to do this or what limitations there might be. Here we examine these assumptions to see what impact they have on performance and whether more robust estimators can be established. This also highlights some of the other information available in the sensor signals.

Specific acceleration

As described above, accelerometers measure the total acceleration of a body with respect to the inertial frame. On a moving animal at the earth's surface, the measured acceleration is the sum of the specific acceleration due to thrust, drag, lift, and buoyancy, and as well as the gravitational acceleration. That is, the accelerometer signal is a mixture of the orientation-related gravity signal defined in the previous section, and signals due to the kinematics of swimming and maneuvering. It is an essential point that these signals cannot be separated without further information.

The specific accelerations acting on the tag can be grouped together in a time-varying 1×3 vector, S_t^t . (this notation means the specific acceleration in the tag frame at time t). The components of this vector are the surge (x-axis), sway (y-axis) and heave (z-axis) accelerations. Including specific acceleration, the accelerometer in the tag frame will actually measure: $A_t^t = -g.[0 \ 0 \ 1]\mathbf{Q}_t\mathbf{W}^{-1} + S_t^t$. Here we are denoting explicitly any quantity that changes with time to remind us that the animal's movements can affect it's orientation as well as its specific acceleration. We convert A_t^t to the animal frame by multiplying by \mathbf{W} :

$$A_t^a = -g.[0 \ 0 \ 1]\mathbf{Q}_t + S_t^t\mathbf{W} = -g.[0 \ 0 \ 1]\mathbf{Q}_t + S_t^a$$

where S_t^a is the specific acceleration in the animal frame. Using the Euler angle decomposition of \mathbf{Q} , this is equal to:

$$A_t^a = [-g.\sin(p_t) + s_{x,t}^a \quad -g.\cos(p_t)\sin(r_t) + s_{y,t}^a \quad -g.\cos(p_t)\cos(r_t) + s_{z,t}^a]$$

where $s_{x,t}^a$ etc are the components of the vector S_t^a . Several conclusions can be drawn from this:

(i) The estimators of pitch and roll derived above may be biased and inaccurate when there is specific acceleration. For example, the pitch estimator is now:

$$\hat{p} = -\text{asin}(a_x^a / \|A^t\|) = \text{asin}((g \sin(p) + s_x^a) / \|A^t\|)$$

which is dependent on the specific acceleration in all of the animal axes (via $\|A^l\|$). A similar result holds for the roll estimator. Because these estimates are needed to compute the heading, the heading estimate will also be less accurate and possibly biased when there is specific acceleration.

(ii) The magnitude of A^a , i.e., the net acceleration experienced by the animal, is no longer equal to g . This can be seen by computing $\|A^a\|^2$, the square of the norm of A^a , which is equal to the sum of the squares of the components in A^a :

$$\begin{aligned}\|A^a\|^2 &= A^a A^{aT} \\ &= (-g \cdot [0 \ 0 \ 1] \mathbf{Q} + S^a) (-g \cdot [0 \ 0 \ 1] \mathbf{Q} + S^a)^T \\ &= g^2 - 2g [0 \ 0 \ 1] \mathbf{Q} S^{aT} + \|S^a\|^2\end{aligned}$$

Thus the net acceleration can be greater or less than g , depending on the direction of the specific acceleration S^a . This provides a helpful indication of times when there is a lot of specific acceleration: if $\|A^a\|$ is not close to g , then the animal is accelerating (or being accelerated) and the orientation estimates may be untrustworthy. We can also use the magnitude of A^a (or equivalently of A^l) to get bounds on the magnitude of the specific acceleration, $\|S^a\|$. Re-ordering the equation for A^a gives:

$$S^a = A^a - g \cdot [0 \ 0 \ 1] \mathbf{Q}$$

Taking the norms of both sides:

$$\|S^a\|^2 = \|A^a\|^2 + 2g [0 \ 0 \ 1] \mathbf{Q} A^{aT} + g^2$$

The maximum absolute value of the term $2g [0 \ 0 \ 1] \mathbf{Q} A^{aT}$ in the above equation is $2g \|A^a\|$ but the sign of the term can be positive or negative depending on the direction of the specific acceleration. Thus:

$$\|S^a\|^2 \leq g^2 + 2g \|A^a\| + \|A^a\|^2 = (\|A^a\| + g)^2$$

and

$$\|S^a\|^2 \geq g^2 - 2g \|A^a\| + \|A^a\|^2 = (\|A^a\| - g)^2$$

So: $|\|A\| - g| \leq \|S^a\| \leq \|A\| + g$.

That is, given the magnitude of the measured accelerometer signal, $\|A\|$, the magnitude of the specific acceleration in this signal will be between $|\|A\| - g|$ and $\|A\| + g$. For example, if the measured acceleration magnitude is 16 m/s^2 , the specific acceleration must be between 6.2 and 25.8 m/s^2 . These are admittedly fairly broad limits but better than nothing.

The above easily calculated limits on $\|S^a\|$ can be used to check for times when the specific acceleration is large giving unreliable orientations. As a rule of thumb, if the specific acceleration is more than about $1\text{-}2 \text{ m/s}^2$, i.e., $10\text{-}20\%$ of g , the orientation estimates can start to get untrustworthy (depending of course on the precision you are expecting). While you may not care about moderate errors in the pitch or roll of a maneuvering animal, even small errors in the orientation will accumulate in the dead-reckoned track and lead potentially to large errors in the position of the animal. As orientation errors increase with animal movement, the errors will be greatest when the animal is maneuvering, surging or braking, suggesting that the dead-reckoned track may be least reliable for active behaviours like

foraging.

The magnitude of the specific acceleration depends on the position of the accelerometer on the animal, as well as the size of the animal and its locomotory rate and amplitude. To show this, think about a whale fluking. The body of the animal moves up and down as the propulsive wave generated by the flukes travels up the body (Fish 1993). The amplitude of body displacement decreases with distance from the flukes so a tag attached near the flukes will experience large vertical displacements while a tag near the rostrum will move much less. If the peak displacement at the tag position is d meters and the fluking rate is f Hz, the displacement of the tag in the ventro-dorsal axis as a function of time is approximately $d \cdot \sin(2\pi ft)$. The heaving acceleration at the tag location is the double differential of this with respect to time, i.e., $-d \cdot (2\pi f)^2 \cdot \sin(2\pi ft)$. The peak heaving acceleration is then $d \cdot (2\pi f)^2 \text{ m/s}^2$. For a large whale with $f=0.2\text{Hz}$, the peak heave is $1.6d \text{ m/s}^2$. Even if the tag is attached near the flukes where d could be perhaps 0.5 m , the peak heave will be 0.8 m/s^2 which is 8% of g so orientation errors during straight-line swimming should be fairly small. However, for a dolphin fluking at 1 Hz , the peak heave is $39d \text{ m/s}^2$. If the tag is attached near the flukes where d could be about 0.1 m , the peak heave will be 3.9 m/s^2 which is 40% of g so orientation will be unreliable even during regular swimming, let alone fast maneuvers. Thus, a tag attachment point close to the rostrum is needed on a small fast-swimming animal to minimise orientation errors due to heave. Alternatively, if the objective is to measure specific acceleration during fluking (e.g., in estimating energetics) rather than the animal's orientation, a caudal tag attachment is needed to maximise this signal in the accelerometer over the orientation signal.

The above example illustrates that swimming motions can produce large specific accelerations if the animal strokes at a high rate and has a high tail displacement. A similar result could probably be obtained for a flying animal. Stroking rate scales roughly with the inverse of body length (Sato et al. 2007) and body displacement should also scale with length so peak heave acceleration will scale approximately inversely with animal size, i.e., small animals will produce higher heave accelerations than larger animals. This is because heave acceleration increases with the square of stroke rate. As a result, specific acceleration during locomotion will likely dominate accelerometer readings on small animals but will represent a small part of the signal on large animals. This scaling effect should be considered carefully when designing experiments especially when performing comparative studies on animals with different body sizes.

It is often assumed that animals tend to change orientation slowly, at least with respect to their stroke rate, while specific accelerations are produced at the stroke rates and above. If this holds over a wide enough range of behaviour, it suggests that the orientation and specific acceleration components in the accelerometer signal can be separated by filtering (Wilson et al. 2006). A low-pass filter with cut-off frequency set to below the fluking rate could then be used to isolate the orientation signal while a complementary high-pass filter would isolate the specific acceleration. Although it seems reasonable, the assumption that specific and gravity accelerations can be separated spectrally has yet to be shown experimentally that I am aware of but there is at least one general counter-example. The swimming motion of marine animals involves vertical (for whales) or horizontal (for seals and many fish) displacements of the body to support beating of the propulsive surface, as discussed above. This displacement moves in a diminishing wave up the body effectively rotating the body in a pitching (whales) or yawing (seal and fish) motion. Yawing motions in a seal or fish will not be observed by an accelerometer if the animal is horizontal because an accelerometer does not detect heading changes. Likewise, pitching motions of a cetacean will not be observed if it is swimming on its side, i.e., with a roll of $\pm 90^\circ$. However, in every other orientation of these varied taxa, the accelerometer will register

orientation changes due to locomotion. These orientation changes occur at the stroking rate and so would be incorrectly grouped with the specific acceleration by a high-pass filter. If the amplitude of the pitching or yawing motion is $\pm 10^\circ$, the worst case signal variation in an accelerometer axis will be $\pm 1.7 \text{ m/s}^2$ which could be comparable or larger than the heave acceleration for a large animal.

Thus, specific acceleration and orientation cannot be separated in general by filtering the accelerometer output, although the errors committed in doing so may only be substantial for large animals. However, a broader objective of filtering may be to separate the faster more energetic movements from the slower navigational movements be they orientation changes or specific accelerations. A simple but effective way to achieve this is to differentiate the accelerometer outputs. The differential of acceleration is termed jerk (units: m/s^3). For digitally sampled data, differentiation can be approximated by the difference from sample-to-sample, multiplied by the sampling rate, i.e., $J_k^a \approx f_s \cdot (A_k^a - A_{k-1}^a)$. Differentiating the accelerometer signal is the same as applying a single pole high-pass filter. Differentiation emphasises rapid changes in orientation and specific acceleration while removing the slowly-changing mean orientation. Because orientation and specific acceleration cannot be separated by filtering, J_k is not the specific jerk (i.e., the differential of the specific acceleration) but rather represents all fast movements experienced and made by the animal. Jerk is a vector and so can be defined in the animal or tag frame by using A^a or A^t , respectively, in the formula above. The norm of the jerk, $j_k = \|J_k^a\|$, being a single number at each sample, may be more useful to summarize animal activity. This measure is independent of the orientation of the tag on the animal and so $\|J_k^a\| = \|J_k^t\|$.

Calibration errors

So far, we have assumed that the accelerometers and magnetometers are perfectly calibrated and read in units of m/s^2 and μT , respectively, with no scale or offset errors. In practice, the scale and offset of each sensor would be determined at the factory or before a deployment using bench calibrations. Accelerometers are straightforward to calibrate due to the constant gravitational field. Magnetometers are more difficult to calibrate requiring a Helmholtz coil and a magnetically-quiet location (most offices and laboratories have complex and variable magnetic fields due to electrical cables and equipment).

However, there are a number of reasons why bench calibrations may not fully characterize the behaviour of a sensor in the field. The sensitivity and offset of MEMS accelerometers varies with temperature and hydrostatic pressure with the latter being a major source of error for deep-diving animals. Pressure sensitivity can be as much as 5 m/s^2 per 1000m of depth (i.e., the accelerometer signal can be offset by $g/2$ at 1000m depth) but this varies widely with manufacturer and even by axis of a tri-axial sensor. Magnetometers are influenced by temperature, the presence of close-by ferrous materials (including the nickel-plated pins on electronic components near the sensor) and magnets. Magnetometers also have a memory effect in which exposure to strong magnetic fields reduces their sensitivity until a degaussing operation is performed. Many tags are disabled to save battery power when not in use by placing a magnet over a reed-switch. This strong magnetic field is enough to change the calibration of a magnetometer.

Many of these effects are difficult to characterize in bench calibrations. Combined effects such as pressure and temperature are especially difficult to establish without special test facilities. The possibility of using the data collected from sensors attached to an animal to deduce the calibration is therefore of great interest. There is a sound basis to this idea: in the absence of specific acceleration, the accelerometer vector should have a magnitude of $g \text{ m/s}^2$ but a direction that depends on the orientation

of the animals. This means that the accelerometer vector (either in the tag or animal frame) should describe a point lying on a sphere centered at the origin with radius g . If all the accelerometer vectors recorded during a deployment are plotted on a 3-dimensional display, they should draw a part of the surface of this sphere. If the sphere is squashed in one or more directions, it indicates a scale factor error in those axes of the sensor. If the center of the sphere is offset from $[0,0,0]$, the sensor outputs are offset by the same amount. If the offset is a function of pressure or temperature this will be evident by plotting $\|A^t\|$ against depth or temperature (assuming that the tag has these sensors).

The magnetometer vectors also have a spherical distribution except that the sphere radius should be b μ T, the magnetic field intensity at the tagging location. Thus, errors in accelerometers and magnetometers can be detected and calibrated out using data collected in the field. Some algorithms for data-driven calibration of magnetometers and accelerometers are described in Gebre-Egziabher & Elkaim (2006) and Camps et al. (2009). There are two important caveats with data-driven calibration:

- (i) Data-driven calibration only appears to be practical for tri-axial sensors and this is a good reason to use a full set of sensor axes in a tag even if this requires a little more power consumption.
- (ii) The animal must change orientation sufficiently throughout the deployment for the vectors to map out a reasonable amount of the sphere. If this is not the case, data-driven calibration may result in a completely incorrect solution. In practice this only seems to be a problem with short data sets on resting animals but some species may pose greater difficulties in this regard.

Sensor frequency response and digital sampling

Ultimately, all sensors in a tag are sampled digitally and the results are stored in memory or telemetered. The digital version of the signal will be a faithful copy of the sensor output provided that aliasing effects are excluded by:

- (i) low-pass filtering the sensor output so there is minimal energy above a frequency f_u , and
- (ii) sampling the filtered signal at $f_s > 2f_u$.

The low-pass filter, which must be an analog filter placed prior to the analog-to-digital convertor in the acquisition circuit, is often called an anti-alias filter. If an adequate anti-aliasing filter is not used, signals with energy above $f_s/2$ will be aliased meaning that they will appear at lower frequencies in the digitized signal and it will be impossible to recover the original signal. For example, accelerations due to stroking at 1.5 Hz, will appear at a stroke rate of 0.5 Hz in data sampled at 2Hz if an anti-alias filter is not used. If a filter is used, the digitized signal will show no stroking which is the correct result: there is no stroking in the frequency range acquired by the tag. Scientifically, this is a far better result than reading an ambiguous stroke rate. For this reason anti-aliasing filters should be considered essential for any sensor which will be subject to temporal or spectral analysis.

Accelerometers and magnetometers usually have a frequency response extending to at least 1kHz. This means that these sensors can resolve very rapid movements provided that the digital sampling rate is sufficiently high. However, to save memory and extend the life of tags, sensors are often sampled at quite low rates. This means that there can be substantial high frequency energy in sensor signals which will alias if an adequate filter is not used. Even if the animal to which the tag is attached is not capable of stroking above $f_s/2$, there are other high frequency signals that can alias, for example, fast prey capture movements and electrical noise in the sensor. The best practice is always to include an adequate anti-alias filter and this will also result in the cleanest, most reliable signals. Given the capacity of

semiconductor memory now, there is little reason to sample sensors at low rates. Tags with high sensor sampling rates are revealing high-frequency signals related to foraging in several taxa providing an exciting indication of the increased range of behaviour that can be sampled by wide bandwidth devices.

Data from different sensors are often combined during data analysis. An example is the combination of pitch and roll, derived from an accelerometer, with magnetometer signals to deduce heading. Care must be taken when combining signals from different sensors to ensure that the data represent identical times and have compatible bandwidths. This is particularly difficult to ensure if the sensors are sampled at different rates or, even worse, if they are sampled asynchronously as is the case if multiple tags are used on the same animal. Artifacts resulting from poor tag design and analysis decisions may be misleading and can reduce the interpretive power of the data (Shiomi et al. 2010).

Tag orientation on the animal

Some types of tags can be attached to free-swimming animals without the requirement to capture and restrain the animal. These tags might be delivered ballistically or with a long pole, and attach to the animal using suction cups (for cetaceans) or skin-penetrating barbs. There is usually little control over where on the animal these tags attach or the orientation of the tag on the animal. In the case of suction cup attachments, the tag may also slide over time and so change its orientation. For these types of attachment, it is necessary to estimate \mathbf{W} , the rotation matrix relating the tag and animal frames, before the animal orientation can be deduced. Photos of the tag on the animal can be helpful for this but the orientation can also be inferred from the sensor data for some taxa making it practical to track changes in the orientation over the deployment.

Estimation of \mathbf{W} from sensor data relies on the identification of intervals in the data when the orientation of the animal is known, at least in the mean. A good example is surfacing intervals in air-breathing marine animals. Many species rest or swim slowly at the surface while breathing and these intervals can be detected readily with pressure or water conductivity sensors. The mean orientation of animals in breathing intervals will be constrained by the need to maintain the breathing apparatus out of the water, for example, for cetaceans, the mean pitch and roll should be close to 0, implying that the mean animal frame accelerometer vector should be $[0\ 0\ -g]$.

As with any rotation matrix, \mathbf{W} or \mathbf{W}^{-1} can be decomposed into yaw, pitch and roll components. For \mathbf{W}^{-1} , these components refer to the set of rotations needed to align the axes of the animal to the axes of the tag. For convenience, in the following, we will estimate \mathbf{W}^{-1} , as opposed to \mathbf{W} , and then just invert the matrix. For cetaceans, the pitch and roll of the animal with respect to the tag can be deduced by comparing the mean accelerometer vector, \bar{A}^t , measured on the tag during breathing intervals to the expected animal frame vector, $[0\ 0\ -g]$, i.e., by finding p_0 and r_0 that solve:

$$\bar{A}^t = [0, 0, -g] \mathbf{W}^{-1} = [0, 0, -g] \mathbf{Y}(\gamma_0) \mathbf{P}(p_0) \mathbf{R}(r_0)$$

Here, γ_0 , p_0 and r_0 are the Euler angles defining \mathbf{W}^{-1} . As $[0\ 0\ -g] \mathbf{Y}(\gamma_0) = [0\ 0\ -g]$, this gives:

$$\bar{A}^t = [0, 0, -g] \mathbf{P}(p_0) \mathbf{R}(r_0)$$

Substituting the definitions for the rotation matrices \mathbf{P} and \mathbf{R} , we get:

$$\bar{A}^t = -g[\sin(p_0), \cos(p_0)\sin(r_0), \cos(p_0)\cos(r_0)]$$

i.e.,
$$p_0 = \text{asin}(\bar{a}_x) \quad , \quad r_0 = \text{atan}(\bar{a}_y / \bar{a}_z) \pm 180$$

Only one of the several possible values for r_0 will solve the equation so each value should be checked until the correct one is found. Similar results can be derived for animals such as seals that breath in other orientations.

To deduce the yaw, γ_0 , of the tag on the animal, another assumption is required about the movements of the animal. Visual observations of cetaceans suggest that they usually maintain a roll of zero when they dive away from the surface, at least for the first few seconds of the dive. If the tag is yawed with respect to the animal, the steep pitch angle at the start of the dive will be translated into a lower angle pitch and a roll in the tag measurement. Thus, if the accelerometer, after correction with the above p_0 and r_0 values, shows a sudden change in roll at the start of dives, this indicates an uncorrected yaw.

The animal frame accelerometer value for a roll-free dive will look like $g[\sin(p_d) \ 0 \ -\cos(p_d)]$ where p_d is the diving pitch angle. The exact value of p_d doesn't matter - more important is that the 2nd component of the acceleration is zero. We can use the p_0 and r_0 values obtained so far to partially correct the tag measurement during the dive, i.e.,

$$A^{pr} = A^t \mathbf{R}(r_0)^{-1} \mathbf{P}(p_0)^{-1}$$

where A^t is the tag frame acceleration measured at the start of the dive (this could be the mean value over several stroke cycles). The pitch- and roll-corrected acceleration, A^{pr} , should be within a yaw of the correct animal frame acceleration vector, i.e.:

$$g[\sin(p_d), \ 0, \ -\cos(p_d)] = A^{pr} \mathbf{Y}(\gamma_0)^{-1}$$

The yaw, γ_0 , can be calculated by yawing A^{pr} until the 'y' component is zero, matching the animal's assumed acceleration, i.e., so that:

$$A^{pr} \mathbf{Y}(\gamma_0)^{-1} [0 \ 1 \ 0]^T = 0$$

This gives the solution:

$$\gamma_0 = -\text{atan}(a_y^{pr} / a_x^{pr})$$

where the numbers in the atan are the y and x component of A^{pr} . The quadrant of the yaw may need to be adjusted for the animal frame accelerations to make sense, e.g., for the pitch of a diving animal to be negative.

Together, γ_0 , p_0 and r_0 define the rotation matrix \mathbf{W}^{-1} that converts the animal frame to the tag frame. To perform the opposite rotation, \mathbf{W} is required and this can be estimated by $(\mathbf{Y}(\gamma_0)\mathbf{P}(p_0)\mathbf{R}(r_0))^{-1}$. In essence, this method takes advantage of two different stereotypical postures that are performed fairly regularly by the tagged animal to deduce the orientation of the tag on the animal. The method could be

adapted readily to other postures and taxa.

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Glossary

Inertial frame An earth-fixed reference frame used to describe the position or track of an animal in 3-dimensions with respect to a fixed '0' point. Several types of inertial frames are popular depending on if the animal moves over large distances or not (i.e., requiring consideration of the shape of the surface of the earth). The axes of the inertial frame could be [latitude,longitude,altitude] or [north,east,depth] (with respect to a defined point at sea-level) etc.

Navigation frame An animal-centred reference frame used to describe the orientation and relative movements of an animal in 3-dimensions. This is a locally-level frame anchored to the animal. The axes of the navigation frame can be [north,east,up] (left-hand rule) or [north,east,down] (right-hand rule). North usually refers to true-north as opposed to magnetic north.

Animal frame An animal-centred, animal fixed reference frame used to describe the relative movements and orientation changes of an animal in 3-dimensions with respect to its current position and orientation. The axes of the animal frame are [caudo-rostral, left-right, ventro-dorsal] (left-hand rule).

Tag frame A tag-centred, tag fixed reference frame used to describe the measurements made by a vector sensor (e.g., a tri-axial accelerometer) in a tag. The axes should be chosen to match the orientation of the sensor in the tag. For simplicity, we assume that the sensor axes coincide with the obvious physical axes of the tag where X^t is the back-front axis, Y^t is left-right and Z^t is bottom-to-top.

A_t^t Acceleration vector in the tag frame as measured by a tri-axial accelerometer at time t . This is a 1x3 vector: $A_t^t = [a_{x,t}^t, a_{y,t}^t, a_{z,t}^t]^T$ where $a_{x,t}^t$ is the acceleration measured in the tag's x-axis at time t , etc. The units are m/s^2 .

A_t^a Acceleration vector in the animal frame obtained by rotating the tag-frame measurement A_t^t . A_t^a is related to A_t^t by the rotation matrix \mathbf{W} , i.e., $A_t^a = A_t^t \mathbf{W}$.

M_t^t Magnetic field vector in the tag frame as measured by a tri-axial magnetometer at time t . This is a 1x3 vector: $M_t^t = [m_{x,t}^t, m_{y,t}^t, m_{z,t}^t]^T$ where $m_{x,t}^t$ is the magnetic field strength measured in the tag's x-axis at time t , etc. The units are micro-Teslas (μT).

M_t^a Magnetic field vector in the animal frame obtained by rotating the tag-frame measurement M_t^t . M_t^a is related to M_t^t by the rotation matrix \mathbf{W} , i.e., $M_t^a = M_t^t \mathbf{W}$.

g The magnitude of the gravity vector. To a very good approximation the gravity vector has a constant magnitude and points towards the centre of the earth, no matter where the animal is. The vector is then $-g\mathbf{U}$ in a left-hand rule navigation frame, where $g = 9.8 \text{ m/s}^2$ and \mathbf{U} is the up vector.

B The earth's magnetic field vector in the navigation frame. \mathbf{B} varies widely with the location of the animal on the earth. Magnetic field charts and calculators are available on the internet and these usually parameterize \mathbf{B} in terms of intensity, b (micro-Teslas, μT), inclination angle, i , and declination angle, d . Using these values, \mathbf{B} is (in left-hand rule):

$$\mathbf{B} = b [\cos i \cos d, \cos i \sin d, -\sin i]^T \text{ in } \mu\text{T}$$

If an animal remains within a region of a few kilometers, \mathbf{B} can be considered constant to first order. In Hobart, $b \approx 62.5 \mu\text{T}$, $i \approx -70^\circ$, and $d \approx 14^\circ$, so $\mathbf{B} \approx [20.7, 5.2, 58.7]^T \mu\text{T}$ in the left-hand rule.

\mathbf{Q}_t The rotation matrix that relates the animal frame to the navigation frame. This defines the orientation of the animal with respect to the navigation frame and so changes continuously with time.

\mathbf{W} The rotation matrix that relates the tag frame to the animal frame. This is normally constant and independent of the whales movements. It depends entirely on how the tag is oriented on the whale. As this may change with time, e.g., if the tag slides or is rubbed, \mathbf{W} may be a step-wise function of time.

Rotation matrix A 3x3 matrix that has the property of rotating a vector without changing the magnitude of the vector. Rotation matrices are orthogonal, i.e., if \mathbf{F} is a rotation matrix, then $\mathbf{F}^{-1} = \mathbf{F}^T$, i.e., the inverse is equal to the transpose. The inverse (or transpose) of a rotation matrix 'undoes' the transformation, i.e., rotates in the opposite direction.

Norm The norm of a vector is a measure of its magnitude. There are many different ways to describe the magnitude of a vector. The conventional norm is the '2-norm' which is the square-root of the sum of the squares, i.e., for $\mathbf{V} = [v_x, v_y, v_z]$, $\|\mathbf{V}\| = \sqrt{(v_x^2 + v_y^2 + v_z^2)}$.

r The roll angle in degrees. Roll is defined in the animal frame as a rotation around the caudo-rostral axis with a positive roll being a clockwise rotation looking rostrally along the axis. Roll angle can be used to describe the orientation of an animal (i.e., with respect to a dorsal-upwards orientation) or can refer to a change in orientation.

p The pitch angle in degrees. Pitch is defined in the animal frame as a rotation around the left-right axis with a positive pitch being a clockwise rotation looking towards the right along the axis. Pitch angle can be used to describe the orientation of an animal (i.e., with respect to a horizontal caudo-rostral orientation) or can refer to a change in orientation.

γ The yaw angle in degrees. Yaw is defined in the animal frame as a rotation around the ventro-dorsal axis with a positive yaw being an anti-clockwise rotation looking dorsally along the axis. Yaw and heading refer to similar aspects of orientation but yaw is used to describe a change in orientation while heading describes the orientation with respect to north.

h The heading in degrees. Heading is defined in the navigation frame as the compass angle of the animal's caudo-rostral axis. See yaw.

s The forward speed of the animal. This could be either the speed over ground or the speed through water and this must be defined.

V The velocity vector of the animal describing its speed and direction of motion. This is usually defined in the navigation frame and so can be used to estimate (dead-reckon) the position of the animal after a time has elapsed (i.e., new position = old position + V x elapsed time).

Specific acceleration, S_t This is a 3-component vector describing the accelerations created or experienced by the animal which will result in movement, as opposed to the gravity-related acceleration. The 3 components of the vector are referred to as heave, sway and surge. Specific acceleration cannot be measured directly but can be estimated under some circumstances by high-pass filtering the accelerometer signals. Specific acceleration is sometimes referred to as 'dynamic acceleration' but this term is ambiguous and so should be avoided.

Gravity-related acceleration This is the 3-component acceleration vector that would be measured by an accelerometer on a stationary animal. It defines just the part of the acceleration signal that is due to the orientation of the animal with respect to the gravity vector, not any additional acceleration due to changes in speed or heading. Gravity-related acceleration is sometimes referred to as 'static acceleration' but this term is misleading as orientation can change rapidly during locomotion or maneuvers.

t Time in seconds.

k Sensor sample number in a digital system. Sample k occurs at time $t=k/f_s$.

f_s Sampling rate of a sensor in Hertz, Hz (i.e., samples per second).

J_t The jerk vector defined as the differential (i.e., the rate of change) of the measured acceleration. The units of jerk are m/s^3 . Because the measured acceleration contains both specific and gravity-related accelerations, J_t is not the specific jerk (i.e., the differential of the specific acceleration) but may be quite close to this for some sizes of animals and some movements.

j_t The norm of the jerk vector used as a proxy for animal activity.

I The identity matrix, i.e., $\mathbf{I} = [1 \ 0 \ 0; 0 \ 1 \ 0; 0 \ 0 \ 1]$ in 3-dimensions.

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